On the Analysis and Design of Bracing Connections

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INTRODUCTION

For many years, the methods for the analysis of bracing connections for heavy construction have been a source of controversy between engineers and steel fabricators. Beginning in 1981, the American Institute of Steel Construction sponsored extensive computer oriented research at the University of Arizona(1) to develop a rational analysis method. Since 1981 physical testing has been performed by Bjorhovde(2) and Gross(3) on full size models of gusset, beam, and column. The results of this work have not yet been distilled into a consistent method of design. It is the purpose of this paper to do so.

The AISC has formed a task group with ASCE to propose a design method (or design methods) for this problem. The recommendations arrived at by this Task Group at a meeting in Kansas City, Missouri, on March 13, 1990 are contained in Appendix A. This paper will attempt to justify the recommended design methods based on Models 2A, 3, and 4, and will include discussion of certain other possible models, such as Models 1 and 5. It will be noted that the author is co-chairman of this Task Group. This paper, however, is not the work of the Task Group and the author is solely responsible for its content.

EQUILIBRIUM MODELS FOR DESIGN - CONCENTRIC CONNECTIONS

An equilibrium model for concentric connections is defined here as a model of the beam, column, gusset and brace(s) which make up the connection in which the connection interface forces provide equilibrium for the beam, column, gusset, and brace with no forces in the beam and column other than those that would be present in an ideally pin connected braced frame. In other words, there are no couples induced in the beam and column due to the connection components. Figs. 1 through 5 show the interface forces for equilibrium Models 1, 2A, 3, 4, and 5 respectively. Equilibrium models apply to concentric connections, i.e. those for which all member gravity axes meet at a common "working" point, and to eccentric connections, i.e. those for which all member gravity axes do not meet at a common point. In this latter case, couples are induced in the frame members which must be considered in the design of these members.

This paper is an expanded version of a paper of the same title that is part of the proceedings of the AISC National Steel Construction Conference, Washington, D.C., June 1991, Pages 26-1 through 26-33.
Model 1 - KISS

This is the simplest possible model which is still an equilibrium model. It has been referred to as the "keep it simple, stupid!" model, or the KISS model. It is simple with respect to calculations but it yields very conservative designs as will be shown. Thus it is easy to use and safe, but yields cumbersome looking and expensive connections. This method is not recommended by the AISC/ASCE Task Group and is included here for comparison purposes.

Model 2A - AISC

This model is a bit more complex computationally but yields less cumbersome designs than Model 1, which are still conservative. It is a generalization of the method presented in the AISC book Engineering for Steel Construction (4) and hence will be referred to as the AISC Model. In Ref. 4, only connections to column webs are considered. This was intentional because the AISC Manual and Textbook Committee, which oversees the production of this book, could not (in 1983) agree on a proper method for connections to column flanges. Model 2A is a generalization suggested by the author. It will be shown to be conservative.

Model 3 - Thornton

This method was developed by the author and is capable of producing uniform stress distributions on all connection interfaces. For this reason, it will always produce the greatest capacity for a given connection or the smallest connection for given loads. In the sense of the Lower Bound Theorem of Limit Analysis, it comes closest to giving the true force distribution among the connection interfaces. It will be shown to come extremely close to predicting exactly the failure load of the Chakraborti and Bjorhovde(2) tests. Of the three models thus far considered, it is the most complex computationally, but will yield the most economic and least cumbersome connections. Further discussion of this model can be found in Appendix B of Ref. 3.

Model 4 - Ricker

This method was developed by David Ricker, Vice President of Engineering of Berlin Structural Steel Company of Newington, Connecticut and a member of the AISC/ASCE Task Group. As shown in Fig. 4, the forces at the centroids of the gusset edges are always assumed to be parallel to the brace force. The method is fairly complex computationally, as can be seen from Fig. 4a, probably about as complex as Model 3. The moment M is required because the resultant force on the gusset to beam interface does not necessarily pass through the centroid of the beam to column connection causing a moment
M on this connection which must be considered in design for this to be a true equilibrium model. Note that the moment \( M \), because it is a free vector, can be applied either to the beam to column interface or the gusset to beam and gusset to column interfaces. The choice is the designer's option.

A weakness of this model lies in the "rigidity" of the assigned directions of the gusset interface forces. When the connection is to a column web, the gusset to column interface force is still parallel to the brace force. This means there is a force component on this interface perpendicular to the column web. Since the column web is very flexible in this direction, this model may require that the column web be stiffened to accommodate the force component perpendicular to the web. It will be shown later that the results of Gross'\(^3\) test 3A can not be predicted by this model because the web is not stiffened.

**Model 5 - Modified Richard**

Of the five models presented here, this is the only one which is not solely based on first principles, but rather contains empirical coefficients derived by Richard \(^1\) from extensive computer analysis. As originally presented by Richard, this is an equilibrium model only if the force resultants act at some points on the gusset to beam and gusset to column interfaces other than their centroids. Richard has not defined the interface points where his interface forces act. Since it is standard practice in connection design to refer all forces to the centroid of the connection under consideration, the author has done so and called this method the "Modified Richard" method. The moments \( M_B \) and \( M_C \) of Fig. 5 are required on the gusset edges to transport the Richard interface forces to the interface centroids.

As is the case with Model 1, this model was not recommended by the AISC/ASCE Task Group, but is included here for purposes of comparison.

**ECCENTRIC CONNECTIONS**

Eccentric Connections are those with member gravity axes which do not intersect at a common working point. Instead the working point is usually assumed at the face of the flange of the beam or column or both as shown in Fig. 6. This working point is chosen to allow more compact connections to result. Fig. 7 shows the gusset interface forces usually assumed. These are shears on the gusset edges. Because these shears intersect the brace line at a common point equilibrium of the gusset can be enforced, and it is a true equilibrium model only if the couples induced in the beam and column are considered in the design of the beam and column. Figs. 6 and 7 call this the "classical" case because it was a very commonly used method in the past but presently is rejected by many engineers because of the induced beam and column couples. One of the objects of this paper is to investigate the consequences of use of this method.

It should be noted that models 1, 2A, and 3 all reduce to the classical case if \( e_B = e_C \).
Models 4 and 5 do not reduce to the classical case.

COMPARISON OF MODEL PREDICTIONS WITH PHYSICAL TEST RESULTS

Two sets of data for full scale and \( \frac{3}{4} \) scale physical tests are available to assess the accuracy of failure prediction of the five equilibrium models discussed in the previous section. These are the tests of Chakrabarti and Bjorhovde \(^2\) and those of Gross \(^3\).

Chakrabarti and Bjorhovde Tests

A set of six tests were performed by Chakrabarti and Bjorhovde \(^2\) on the specimens of Fig. 8. Fig. 8 was replicated six times, i.e. for each of two gusset thicknesses \( \left( \frac{1}{8}, \frac{3}{8} \right) \) and three brace angles from the horizontal \( (\phi = 30^\circ, 45^\circ, \text{and } 60^\circ) \). Only the \( \frac{1}{8} \) gusset is treated here because the \( \frac{3}{8} \) gusset specimens exceeded the capacity of the testing frame. In the test frame, the specimen was oriented with column horizontal and bolted to the test frame which was in turn bolted to the laboratory floor, and the beam was vertical with top end free. Thus, this setup is roughly equivalent to a situation in a real building where the brace horizontal component \( (P\cos\phi \text{ of Fig. } 8) \) is passed to an adjacent bay. The force \( P\cos\phi \) is referred to as a "transfer force" and denoted \( T_f \). In the calculations to predict capacity using the five models, the transfer force \( T_f \) for the Chakrabarti/Bjorhovde tests is \( P\cos\phi \) and this is made up of \( H_C \) from the gusset to column connection and \( H_B \) from the beam to column connection. Thus, the beam to column connection for all models will be subjected to \( H_B \) (axial) and \( V_B \) (shear).

Tables 1, 2, and 3 give the results of predicting Chakrabarti/Bjorhovde's failure loads using the five models. Appendix B contains the calculations required to generate the capacities given in Table 2 (45\(^\circ\) case) for Model 3. Calculations for all the capacities of these tables for the other models are similar.

It will be seen from Tables 1-3 that Model 3 very closely predicts the actual failure load. In Tables 1 and 2, it even predicts the correct interface where failure first occurs. Table 3 presents something of an anomaly. In all three tests, the brace to gusset connection was exactly the same and has a predicted capacity (all Models) of 142k. In Tables 1 and 2, the brace to gusset connection failed at 143k and 148k respectively, but in Table 3, the 60\(^\circ\) case with exactly the same brace to gusset connection, it did not fail and a load of 158k was achieved at which the gusset to column connection failed. For some unknown reason, the brace to gusset connection for this 60\(^\circ\) case is much stronger than expected. Note that given a brace to gusset connection with an actual capacity (per
Chakrabarti/Bjorhovde's test) in excess of 158k, Model 3 correctly predicts that the gusset to column connection will fail first at 155k, about 2% less than the actual failure load of 158k.

Further reviewing the data of Tables 1-3, it can be seen that all the models are conservative, Model 3 being barely so, and Model 1 being grossly conservative. The other Models are moderately conservative except that Model 4 sometimes duplicates Model 3's performance. The author believes that the results presented in Tables 1-3 show that all these Models are reasonable for design except Model 1 which is consistently too conservative.

Gross Tests

Gross (3) performed three essentially full scale (they are referred to as \(\frac{3}{4}\) scale in Gross' report) tests on the specimens shown in Figs. 9, 10, and 11. These tests differ from the Chakrabarti/Bjorhovde's tests in that the beam and column(s) participate in the tests by frame action. Fig. 12 shows the complete test specimen (specimen 1) and Fig. 13 gives a schematic of the test frame. In the Chakrabarti/Bjorhovde tests, the brace connection is effectively isolated from the frame action. Therefore, Gross' tests are more realistic as compared to a real structure where frame action cannot be eliminated, and it will be interesting to see how well the equilibrium models predict failure in this case.

Gross' tests differ from Chakrabarti/Bjorhovde in another way: Specimens 2 and 3 are eccentric connections. As noted earlier the five equilibrium models can be used for Specimens 2 and 3, but couples of possibly all three types (see Table 7) will be induced in the members for these configurations.

Tables 4, 5 and 6 compare the model predictions with Gross' actual failure loads. It will be seen that all the equilibrium models are conservative and none, not even Model 3, comes close to predicting the actual failure load of the connection interfaces. The reason for this in the case of Specimens 1 and 2 is due to frame action. This has the effect of tending to reduce certain of the connection interface forces postulated by Models 1-5. As seen in Tables 4 and 5, the controlling interface in almost every case is the gusset to column interface, and the controlling limit state on this interface is clip angle bending (tension capacity) or prying action. Since frame action reduces \(H_c\), the calculated connection capacity based on \(H_c\) unreduced by frame action is too low. However, if we are to use frame action to justify an increased capacity of the connection, we must determine if the connection has an effect on the frame which reduces its capacity. The following sections will investigate this and show that indeed the connections of Specimens 1, 2 and 3 all tend to reduce the capacity of the frame due to increased member moments and reduction of frame sidesway stiffness. If these effects are considered, the Models 1-5 will be seen to be less conservative than they appear to be.

For Specimen 3, there is no frame action because the connection to the column web does not engage the column flanges. Table 6 shows that the test capacity/predicted capacity ratio is closer to one than was the case with Specimens 1 and 2. This is probably because
of the lack of frame action. Table 6 indicates that Models 4 and 5 are not applicable to this Specimen. This occurs because these models require that the column web be capable of resisting normal forces \( H_C \) and couples \( M \) or \( M_C \). Since the web is very flexible, these forces cannot be carried unless the web is stiffened to do so. Since Specimen 3 is not so stiffened there is no path for these forces to follow and so they will not exist. Thus, the models which predicate these forces are inapplicable to Specimen 3. Also, because of the lack of frame action, all of the eccentric moment will be in the beam. At a lateral load of 70 kips, this means a moment equal to \( 70 \times 16 = 1120 \) k-inches in the W16 x 40 beam, which is 28% of the elastic beam moment \( M_Y = F_Y \times S_X = 61 \times 64.7 = 3947 \) k-inches.
TABLE 1

<table>
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<tr>
<th>Model</th>
<th>Brace to Gusset (A) kips</th>
<th>Gusset to Beam (B) kips</th>
<th>Gusset to Column (C) kips</th>
<th>Beam to Column (D) kips</th>
<th>Predicted Capacity kips</th>
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(1) NL = no limit; this part of connection does not carry any of brace load P.
### TABLE 5

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(1) NL = no limit; this part of connection does not carry any of brace load P.
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<th>Gusset to Beam (B) kips</th>
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</table>

(1) NL = no limit; this part of connection does not carry any of brace load P.
(2) NA = not applicable. This method requires column web stiffeners to carry the force.
MEMBER AND CONNECTION FORCES IN BRACED FRAMES

Three types of moments can exist in the members of a braced frame. These are listed in Table 7.

Table 7 Member Moment Types and Causes

<table>
<thead>
<tr>
<th>Member Moment Type</th>
<th>Cause</th>
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<tr>
<td>I. Eccentric</td>
<td>Non-intersection of Member Gravity Axes</td>
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<tr>
<td>II. Distortion</td>
<td>Frame Distortion and Connection Properties</td>
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<tr>
<td>III. Connection</td>
<td>Equilibrium of gusset, beam and column</td>
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</table>

These will be referred to in what follows as moments of Type I, II, or III. Eccentric member moments (Type I) occur only in braced frames with non-concentric connections. The distribution of these moments to the frame members incident to a joint depends on the rigidity and strength of the connection interfaces between the members. Distortion moments (Type II) in members occur in braced frames to the extent that the connection interfaces are rigid (strong). An ideal friction-less pin connected frame would have no distortion moments. Connection member moments (Type III) are induced in the members of braced frames because of connection properties and equilibrium.

Associated with each type of member moment are connection interface forces which will be identified as Types I, II and III also to indicate their source. Type III interface forces need some additional discussion. These are different from the eccentric and distortion forces and are perhaps not commonly understood. Basically, they are the forces which exist on the connection interfaces when each part of the connection, i.e. gusset, beam, and column, is treated as a free body diagram and forced to satisfy equilibrium. The equilibrium models 1, 2A, 3, 4, and 5 of Figs. 1 through 5 show the Type III connection interface forces which will provide zero connection moments (Type III) in the members of concentric frames. If these connection interface forces can not be achieved because of connection flexibility (lack of strength), non-zero connection moments (Type III) will be induced in the members. For eccentric joints, the Type III connection forces are shown in Fig. 7 or in Figs. 1-5 with $e_B = e_C = 0$. In this case, the Type III connection forces will always induce non-zero Type III moments in the members.
FURTHER ANALYSIS OF GROSS' TESTS

SPECIMEN 1.

Gross\(^3\) gives data which allows the calculation of bending moments in the beam and columns at their juncture. Fig.3.24 of Ref. 3 reproduced as Fig. 14 gives the moment diagram for Test 1A. Also shown are the experimental transverse forces which produce these moments. Since Test 1A had all members intersecting at a common gravity axis working point, these moments are of Type II and Type III, i.e. distortion and connection induced moments. If the connections were capable of generating Type III connection forces, the member moments would be of Type II only, and their distribution would be as shown in Fig. 15. The expected distribution would have column moments half the magnitude of the beam moment and of opposite sense. The moments in the columns balance the moment in the beam with no unbalanced couple. Obviously, this is not the case in Fig.14. The difference between Fig. 14 and Fig. 15 must be due to Type III moments, i.e. to connections which cannot generate Type III connection interface forces or to some error in the test data or procedure. As noted by Gross, the top cross head was not as rigid as an equivalent building frame and moved down slightly under lateral load. This may account for some additional small distortions in the test specimen but cannot account for the vast difference between Figs. 14 and 15. It is postulated here and will be demonstrated that the difference between Figs. 14 and 15 can be explained by the presence of Type III moments caused by the absence of connection rigidity (strength) to develop Type III connection interface forces.

Consider the connection interface between the beam and the column. The moment on this interface is 819k-in and is due to the 9.1k beam reaction shown in Fig. 14. Since the braces are known to be able to sustain very little moment, the 819k-in moment is statically equivalent to the connection forces shown in Fig.16. Similarly, Fig. 17 shows the Type III connection forces necessary to provide equilibrium with no Type III moments induced in the beam and columns. These forces are derived from model 2A or 3. Figs. 16 and 17 show that the Type II and III connection interface forces normal to the column flange oppose each other. This will always be true.

Combining the forces of Figs. 16 and 17 we get Fig. 18. These are a possible Type II - Type III combination of forces. Fig.18 is interesting because it shows that the top gusset to column connection sees a net compressive load normal to the clip angles. Equilibrium models 2A, 3, 4, and 5 would all postulate a tensile load here. Also, the bottom gusset to column connection sees a net tensile force whereas the models 2A, 3, 4, and 5 would result in a compressive force. It is clear that in Gross' experimental results, the frame distortion effect (Type II) overpowers the equilibrium (Type III) distribution of forces. This explains why no "prying action" type of failure was observed in the top gusset to column connection as was predicted by the equilibrium models.

Considering prying action, the theory presented in Appendix C of this paper can be used to show that the maximum yield or separation load per bolt for the clips of Gross' specimens is \( T_y = 1.028k \) and the maximum ultimate load is \( T_u = 1.656k \). In Gross' tests, there was no visible distortion or separation between the clip angles and the column flange. Therefore, it is reasonable to assume that the load per bolt did not exceed 1.028k. This being the case, the maximum tensile load that the 8 bolt connection of each gusset to the column flange can carry is \( 8 \times 1.028 = 8.222k \). This means that the 14.02k in Fig.18 between the column and bottom gusset could not have been achieved. A redistribution of forces must have taken place as shown in Fig.19. The final horizontal forces of Fig.19 coupled with the unchanged vertical forces satisfy equilibrium of the beam and column with no couples in the beam or column other than those due to distortion. Also, the clip strength is not exceeded. The final forces of Fig.19 are shown for convenience in Fig.20. These forces are an admissible set for Gross' Test 1A because:
1. They satisfy equilibrium for the beam.
2. They produce the moment and force distribution shown in Fig. 21 which very closely reproduces Gross’ results as shown in Fig. 14.
3. They consider the strength of the beam to column connection clip angles and the observed lack of separation between the clips and the column flange.

The interface forces of Fig. 20 clearly show why no "prying action" or clip bending failure mode was evident in the Gross tests (similar forces can be obtained for tests 2A and 3A). Can it be concluded therefore that clip bending need not be a considered failure mode? The answer is no for several reasons. First, suppose the clip could not sustain any tensile axial force, i.e. instead of 1,028k Bolt, 0k/Bolt. In this case the beam of Fig. 20 can not be sustained in horizontal and couple equilibrium, which is impossible. Also, the 29.75k forces of Fig. 16 could not be developed, which means that the equilibrium forces of Fig. 17 will prevail, but the 12.5k tension force can not be developed. This in turn means equilibrium won't be satisfied, which is impossible. Therefore, the clips must be capable of sustaining some tensile axial force. In this case it can be shown that the least possible tensile capacity is .7714k/Bolt, which means 1/4 clips are required. If 1/4 clips are absolutely required in this case for equilibrium, then heavier clips will be required in many realistic cases and their design cannot be ignored.

Secondly, it is not known if the relationship of the distortion forces of Fig. 16 to the equilibrium forces of Fig. 17 is constant. Are there possible cases where the distortion (Type II) forces are small in relation to the connection (Type III) forces? Gross also makes this point in his report. If the distortion forces are small, then Fig. 17 indicates that the clips would sustain a tensile force of 10-12 kips or 14-15 kips if the loads reverse. Clips 5/16 to 3/8 inch thick would be required to carry these loads without separation.

Even in this case (Test 1A) there is reason to believe that the distortion forces are too big, possibly because of the lack of the ability of the beam to column connections to develop sufficiently large Type III connection forces.

Considering the model used in Gross’ tests, as shown in Fig. 13, it can be shown that an approximation for the moment between the beam and column is given by

\[ M = \frac{6P(h^2 I)}{A [hn]^2} \left( 1 + \frac{I_B}{I_C} \right) \]

where:
- \( P \) = Force in diagonal
- \( h, l \) defined in Fig. 13
- \( I_B \) = Moment of inertia of beam
- \( I_C \) = Moment of inertia of column
- \( A \) = Area of Brace
Also, the sideways angle is given by

\[ \gamma = \frac{P}{AE} \frac{h^2 + l^2}{hl} \]

and the total sidesway displacement is

\[ \Delta = 2h\gamma \]

For the data of Gross' model (Specimen 1) at a lateral load of 70 kips

\[ M = \frac{6 \times 81.63 \left( 54^2 + 90^2 \right) (518)(272)}{6.16 \times (54 \times 90)^2 \left( \frac{2 \times 272}{54} + \frac{518}{90} \right)} = 330 \text{ k-in} \]

and

\[ \Delta = \frac{2 \times 54 \times 81.63 \times 54^2 + 90^2}{6.16 \times 29000 \times (54 \times 90)} = .112" \]

Table 8 compares the theoretical and experimental values of \( M \) and \( \Delta \).

<table>
<thead>
<tr>
<th>Table 8 Theoretical and Experimental Moment and Displacement</th>
<th>Theoretical</th>
<th>Experimental</th>
</tr>
</thead>
<tbody>
<tr>
<td>Beam to Column Moment ( M ) (k-in)</td>
<td>330</td>
<td>819</td>
</tr>
<tr>
<td>Sideway Displacement ( \Delta ) (in)</td>
<td>.112</td>
<td>.26</td>
</tr>
</tbody>
</table>

It will be seen that the experimental results are about \( 2\frac{1}{2} \) the theoretical. One would expect better agreement than this because the theoretical method is the classical slope-deflection method for determining secondary moments in trusses and is known to be very accurate. For instance, a stiffness analysis using a commercially available problem, STAAD III yields a moment of 278k-in. and a displacement of .114". Therefore, the disparity must be due primarily to the weakness of the beam to column connection which allows larger displacements.
and associated distortion moments than would be expected.

It should be noted that Eqs. 1-3 do capture the correct relationship between displacement and moment. If the true displacement of the real structure is $0.26^\circ$, then Eqs. 1-3 give $M = 780$ k-in, which is within 5% of the experimental value of 819k-in, which leads to the third reason why "prying action" should not be ignored. If the beam to column connection is indeed rigid, then the moment between the beam and column for a brace force $P = 81.63^k$ (i.e. $\Delta = 0.112$ inch) is 330k-in and the column moments would be 165k-in, as shown in Fig. 22. Compare Fig. 22 to Fig. 14. The much larger moments of Figs. 14 are due to the weakness of the connection between the beam and column. Can this be ignored? The whole idea of the distortion forces is based on considering the beam to column connection as rigid, i.e. able to develop the couple $M = 330$ k-in or the forces $\frac{330}{26} = 12.7k$. Is it then reasonable to suppose that we can ignore the clip angle tension capacity when such a capacity was assumed to exist from the outset? As noted above, the distortion forces (Type II) and the equilibrium connection forces (Type III) will always oppose each other, and using Eq 1 and Model 2A or 3, we can compute their magnitudes. From Eq.1, the horizontal force between the top and bottom gussets and the column of Gross’ specimen 1 is:

$$H_{C_{II}} = \frac{3P}{e_B + b} \frac{L^2K_{BC}}{hL}$$

where

$$L = \sqrt{h^2 + l^2}$$

$$K_{BC} = \frac{I_{B_{I_{C}}}h}{(\frac{2I_c}{h + l} + \frac{I_c}{h + l})}$$

and the subscript II indicates this force is due to distortion. From Model 2A or 3:

$$H_{C_{III}} = \frac{P}{e_B + b} \frac{h e_c}{L}$$

where the subscript III indicates this is a connection interface force required for equilibrium with no connection induced member couples. If the brace force $P$ is tension, $H_{C_{III}}$ will also be tension and $H_{C_{II}}$ will be compression. Therefore, the net horizontal force is:

$$H_C = H_{C_{III}} - H_{C_{II}}$$

$$= \frac{P L}{e_B + b} \left[ \frac{h e_c}{L^2} - \frac{3LK_{BC}}{hL/A} \right]$$
Eq 6 can be put in dimensionless form as:

\[
\frac{H_c}{P} = \frac{e_c h}{(e_B + h)L} (1 - \eta)
\]

where \( \eta = \frac{3L^3}{h^2} \frac{K_{BC}}{e_A} \)

The quantity \( \eta \) is proportional to \( H_{c_{II}}/H_{c_{III}} \) and so can be used to compare the magnitude of the distortion forces (Type II) as compared to the connection forces (Type III) required for equilibrium with no connection induced member couples. If \( \eta \) is small, the distortion has a small effect. If \( \eta = 1 \) the Type II and Type III balance each other. If \( \eta > 1 \) the Type II (distortion) forces predominate. Table 9 shows the parameters for a variety of bracing connections from light to heavy, with light beams and heavy columns and vice versa, and with beam and column the same size. Generally \( \eta \) is small, i.e. much less than 1. Only for the specimen 1 of Gross' tests does it approach 1. Note that the parameter \( \eta \) of Table 9 is for the geometry of Gross' specimen No.1. This arrangement has 2 diagonals framing into the joint. If only one diagonal were present, as for instance in the arrangement shown in Fig.23, the \( \eta \) values would all be doubled. In general, \( \eta \) will be a function of the arrangement of braces, but the usual range of values will be those given in Table 9 to twice the values in Table 9. Since \( \eta \) is \( \ll 1 \) for some cases, it is not reasonable to depend on Type II and Type III forces cancelling each other in design.

In general, we could design for \( H_c = H_{c_{III}} - H_{c_{II}} \) provided that we also design for \( H_{c_{II}} \) forces alone to provide for the correct distribution of member distortion moments. However, we do know that designing for the Type III connection forces alone, which are generally bigger than the Type II forces, yields safe designs based on applying methods incorporating these forces to predict capacities of Bjorhovde's and Gross' physical test specimens, and is much simpler. See Tables 1 through 6 of this paper. Therefore, it is recommended that these connections be designed for Type III connection forces using Models 1, 2A, 3, 4, and 5 which include prying action and that the distortion forces be ignored, except possibly as noted below:

One place where the opposition of equilibrium (Type III) and distortion (Type II) forces can be used to advantage is in the combination of equilibrium forces and transfer forces. Considering Figs. 2, 3, and 4 for models 2A, 3 and 4, the \( H_c \) force between the beam and the column will always be opposed by a distortional force. In addition to these two forces, there is sometimes specified a "transfer" force or "drag through" force \( T_f \) to carry loads to the next bay. In this case it is reasonable to design the beam to column connection for max \( (H_c, T_f) \) rather than \( H_c + T_f \) even when \( H_c \) and \( T_f \) are of the same sense because \( H_c \) will always be reduced by the distortional force.
<table>
<thead>
<tr>
<th>No.</th>
<th>Source</th>
<th>Beam</th>
<th>Column</th>
<th>Brace</th>
<th>h</th>
<th>f</th>
<th>$I_B$</th>
<th>$I_C$</th>
<th>$K_{BC}$</th>
<th>A</th>
<th>$e_c$</th>
<th>$\eta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Gross Spec. 1</td>
<td>W16 x 40</td>
<td>W10 x 49</td>
<td>W8 x 21</td>
<td>4.5</td>
<td>7.5</td>
<td>518</td>
<td>172</td>
<td>1.83</td>
<td>6.16</td>
<td>5</td>
<td>.786</td>
</tr>
<tr>
<td>2</td>
<td></td>
<td>W24 x 55</td>
<td>W14 x 211</td>
<td>W14 x 68</td>
<td>13.0</td>
<td>25.0</td>
<td>1830</td>
<td>2660</td>
<td>2.59</td>
<td>20.0</td>
<td>7.86</td>
<td>.258</td>
</tr>
<tr>
<td>3</td>
<td></td>
<td>W24 x 76</td>
<td>W14 x 370</td>
<td>2L56 x 6 x 1</td>
<td>15.0</td>
<td>12.0</td>
<td>2100</td>
<td>5440</td>
<td>5.87</td>
<td>22.0</td>
<td>8.96</td>
<td>.2346</td>
</tr>
<tr>
<td>4</td>
<td></td>
<td>W24 x 100</td>
<td>W12 x 65</td>
<td>W10 x 49</td>
<td>17.3</td>
<td>30.0</td>
<td>3000</td>
<td>533</td>
<td>1.59</td>
<td>14.4</td>
<td>6.06</td>
<td>.255</td>
</tr>
<tr>
<td>5</td>
<td></td>
<td>W18 x 106</td>
<td>W14 x 605</td>
<td>W12 x 87</td>
<td>24.0</td>
<td>30.0</td>
<td>1910</td>
<td>10800</td>
<td>2.48</td>
<td>25.6</td>
<td>10.46</td>
<td>.091</td>
</tr>
<tr>
<td>6</td>
<td></td>
<td>W24 x 104</td>
<td>W24 x 104</td>
<td>W14 x 90</td>
<td>24.0</td>
<td>28.0</td>
<td>3100</td>
<td>3100</td>
<td>3.23</td>
<td>26.5</td>
<td>12.03</td>
<td>.003</td>
</tr>
</tbody>
</table>
SPECIMENS 2 AND 3

Gross considers two non gravity axis or eccentric models, specimens 2 and 3. The reported capacities for these two specimens in tests 2A and 3A exceed that of test 1A for the gravity axis or concentric specimen. However, the column and/or beam moments are greater for these eccentric models because they are due to eccentricity moments (Type I) as well as distortion (Type II) and connection interface (Type III) moments. It is not known whether these increased moments could cause a failure, but it is prudent to consider that they could and to design for them. To the author’s knowledge, no one has demonstrated that the Type I moments can be safely ignored. Indeed, the AISC\(^6\)\(^{10}\) specification says that gravity axes shall intersect or the effect of non intersection shall be accounted for. The SJI\(^11\) specification allows some variation from concentricity in joist girders, but this allowance has been justified by tests and analysis. In bracing connections, the variation in members and configurations is greater than in joist girders. However, in bracing connections the lateral load is almost always due to wind or seismic loads, and the AISC specification allows a \(\frac{1}{3}\)rd stress increase in this case. This can often be used to justify ignoring the additional couples induced by eccentric connections.

The results obtained by Gross for Test 3A (Fig.3.27 of Ref.3) deserve some additional comments. This is a connection to a column web with the bracing working points at the beam top and bottom flanges. Fig.3.27 of Ref. 3 clearly shows that the column does not participate in the distribution of couples. This occurs because the gusset connections to the column web are sufficiently removed from the column flanges that the column is not mobilized to participate in accordance with I/L in the moment distribution. The beam effectively takes the entire eccentric moment. This exact phenomenon was predicted by Thornton\(^12\) on the basis of stiffness distributions and is what is assumed to occur in connection design models 2A and 3. Model 4, because it always assumes that the gusset to column connection force resultant is parallel to the brace force, would assume that the column is mobilized to carry moment and would require a connection that distributes the horizontal component of the gusset to column connection forces to the column flanges.
SUMMARY AND RECOMMENDATIONS

The time has come to see if the foregoing discussion can be distilled into a design method for bracing connections which is defensible in terms of first principles is universally applicable, is reasonably conservative, and is not too difficult to use. Of the 5 models, i.e. 1, 2A, 3, 4 and 5, only two, i.e. 2A and 3, satisfy all of the above. Model 1 fails because it is not always "reasonably conservative". Models 4 and 5 fail because they are not universally applicable. Also Model 5 is not based entirely on first principles - it is empirical and thus limited to cases similar to those from which the empirical factors were derived.

While all 5 models have been determined to yield safe designs (Tables 1-6) and are thus deemed acceptable for use, Models 2A and 3 will yield the best results in terms of economy and are therefore the recommended methods. Only these two methods will be included in the following discussion.

For concentric connections, design for the connection interface forces given in Figs. 2 and 3. Ignor distortion effects. This has been shown to be conservative, i.e. see Tables 1-6.

For non-concentric connections with working points located as shown in Figs. 6, 10, and 11, design for the connection interface forces shown in Figs. 2 and 3 with $e_B = e_C = 0$ or for those shown in Fig. 7 which are the same. Ignor distortion effects, but check the frame members (beam(s) and column(s)) for the member moments caused by the force distributions given in Fig. 7 (or Fig. 2 and 3 with $e_B = e_C = 0$). For instance, for Gross' Specimen No.2 as shown in Fig 10, Table 10 gives the member moments which result from Model 2A.

<table>
<thead>
<tr>
<th>Table 10</th>
<th>Member Moments (Type III) Due To Model 2A For Gross Specimen No.2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Member</td>
<td>Member Moment (Type III)</td>
</tr>
<tr>
<td>Top Col.</td>
<td>$(V_C)_{TOP} \times e_C$</td>
</tr>
<tr>
<td>Bot. Col.</td>
<td>$(V_C)_{BOT} \times e_C$</td>
</tr>
<tr>
<td>Beam</td>
<td>$((H_B)<em>{TOP} + (H_B)</em>{BOT}) \times e_B$</td>
</tr>
</tbody>
</table>

The moments of Table 10 will always sum algebraically to equal the eccentric moment on the joints due to the non-intersection of the member of gravity axes. Thus equilibrium for the frame members will be satisfied. Note that the distribution of moments will not be the same as what would occur based on an $I_L$ distribution (i.e. Type I moments). Note also that the usual $I_L$ distribution is based on a rigid beam to column connection which will not
be achieved unless heavy clip angles are used (heavier than $1/4$ inch in Specimen No. 2). Finally note that if $T_L$ moments are used, the joint must be designed for them and the distortion moments (Type II) which result must also be included in the analysis and design. Rather than all this, it is recommended to design the member for the Type III moments of Table 10 in addition to the other loads in these members. If the Type III moments are due to wind on seismic loads and do not exceed 33% of similar gravity moments, they can be ignored. In any case, since these moments in the members are in equilibrium with the eccentric moment caused by non-intersection of gravity axes, we know our design will be safe from an ultimate strength point of view because of the Lower Bound Theorem of Limit Analysis.

Finally, for non-concentric connections with working points located at points other than those shown in Figs. 6, 10, and 11, the quantities $e_B$ and $e_C$ of Figs 2 and 3 can be used to locate these points and the analysis of the foregoing paragraph holds with suitable adjustment.
ON DUCTILITY DEMAND AND SUPPLY

DEMAND

In continuum theory, compatibility is the lack of gaps, tears, and dislocations in the continuum. Mathematically, compatibility is enforced by insuring that two sided derivatives of the displacement field with respect to the spatial coordinates of the field exist everywhere within the continuum. Where these derivatives fail to exist, a tear, gap, or dislocation is indicated in the field and at these locations high stresses will result. Because of these high stresses, the ductility demand at these points will be high also.

Conversely, if compatibility is satisfied, displacement derivatives will exist everywhere and stress magnitudes and associated ductility demand will be reduced accordingly.

For the various equilibrium models proposed here, how can we know which gives rise to the least ductility demand? As a possible approach to an answer to this question, consider the Lower Bound Theorem of Limit Analysis. This basically states that

"If an equilibrium distribution of stresses (internal forces) can be found which balances the applied load and is everywhere less than or most equal to the yield stress, then the applied load is less than or at most equal to the plastic limit (failure) load."

If the load achieved is in fact the limit load, then the Lower Bound Theorem has achieved the true failure load. When the true failure load is achieved, compatibility as well as equilibrium and yield is satisfied. Therefore, it is reasonable to postulate that among the various models, the one which predicts the greatest failure load also comes closest to satisfying compatibility, and thereby reducing ductility demand. Of the various models, Model 3 will always predict the greatest load capacity because it is capable of achieving uniform stress distributions over all connection interfaces, i.e. interfaces between the gusset and column, the gusset and beam, and the beam and column.

SUPPLY

Regardless of which model is used to calculate the equilibrium forces on the connection interfaces, it must be recognized that the ductility supplied by the actual connection is not unlimited as assumed by the Lower Bound Theorem of Limit Analysis. Before the equilibrium force distributions are achieved by redistribution, peak stresses will occur which could cause progressive failure by propagation of a fracture occurring somewhere within the connection. To mitigate this possibility, the following suggestions...
are made:

1. Welds between rigid elements such as the fillet weld between the gusset and beam flange will experience a peak stress about 1.5 times the average stress before the ultimate load is achieved. This has been demonstrated by Richard in extensive computer simulations of bracing connections. Therefore, if a uniform distribution of weld stress is realized, as for instance from Model 3, design the connection for 1.5 times the average stress. If a nonuniform distribution of weld stress is realized, i.e. from Model 2A or 4, compare the peak stress to the average stress. If the peak stress is greater than 1.5 times the average, no increase is required. If not, increase the peak stress to 1.5 times the average and design for this.

2. Welds between elements where flexibility exists, such as the welds of the clip angles to the gusset where the bolted leg provides flexibility, no factor is required provided $\alpha > 0$, i.e. the bolted leg is flexible. If $\alpha \leq 0$ design weld as in rigid element case.

3. Bolts and Clip Angles - Design clip thickness such that $\alpha > 0$ so that some flexibility is provided to allow redistribution. If $\alpha \leq 0$, treat the same as welds in paragraph 1 above.

4. Gusset - No special consideration need be given to the gusset itself because its connections to the beam and column provide the necessary ductility.

It will be seen from the above discussion that the models tend to supply the ductility needed. That is, all models other than Model 3 generally predict nonuniform interface stress distributions which give rise to larger welds, clip angles, and bolts. These larger welds, clip angles, and bolts have the capacity to handle the peak stresses that occur before the stresses even out, as they eventually will as predicted by Model 3.
REFERENCES:


Fig. 1  THE SIMPLEST EQUILIBRIUM MODEL AND INTERFACE FORCES - MODEL 1
Fig. 2A - MODEL 2A - AN EXTENSION OF THE AISC® METHOD
Fig. 2B CONNECTION INTERFACE FORCES FOR MODEL 2A
Fig. 3A MODEL 3 - UNIFORM CONNECTION INTERFACE FORCE MODEL PROPOSED BY THORNTON
Fig. 3B CONNECTION INTERFACE FORCES FOR MODEL 3

\[ H_B = \frac{\alpha}{r} P \]
\[ V_B = \frac{e_B}{r} P \]
\[ V_C = \frac{\beta}{r} P \]
\[ H_C = \frac{e_C}{r} P \]
\[ r = \sqrt{\left(\alpha + e_C\right)^2 + \left(\beta + e_B\right)^2} \]
Couple $M$ is required to satisfy equilibrium of beam and column.

Fig. 4A MODEL 4 - PARALLEL INTERFACE FORCE MODEL PROPOSED BY RICKER
\[ e_1 = (e_c + \alpha) \cos \theta - e_B \sin \theta \]
\[ e_2 = (e_B + \beta) \sin \theta - e_c \cos \theta \]

\[ P_B = \frac{e_2}{e_1 + e_2} P; \quad P_c = \frac{e_1}{e_1 + e_2} P \]

\[ M = \frac{e_2}{e_1 + e_2} P (\alpha \cos \theta - e_B \sin \theta) \]

Fig. 4B  CONNECTION INTERFACE FORCES FOR MODEL 4
Fig. 5A  MODEL 5 EMPIRICAL INTERFACE FACE MODEL
PROPOSED BY RICHARD\(^{(1)}\)  (MODIFIED BY THORNTON)

Couples \(M_B\) and \(M_C\) are required to satisfy equilibrium of the gusset, beam and column.
\[ \lambda = 1.4 \left( \frac{\alpha}{\alpha + \beta} \right) - 0.1 \]
\[ \phi_B = 0.6 \phi \quad \text{if } \phi \leq 45^\circ \]
\[ \phi_B = 27 + (8.5 - 20 \frac{\alpha}{\alpha + \beta})(45 - \phi) \quad \text{if } \phi > 45^\circ \]
\[ P_B = \lambda P \quad V_B = P_B \sin \phi_B \quad H_B = P_B \cos \phi_B \]
\[ V_C = P(\sin \phi - \lambda \sin \phi_B) \quad H_C = P(\cos \phi - \lambda \cos \phi_B) \]
\[ M_B = \lambda P(\alpha \sin \phi_B - e_B \cos \phi_B) \]
\[ M_C = P(e_C \sin \phi - (e_B + \theta)(\cos \phi - \lambda \cos \phi_B)) \]

Fig. 5B  CONNECTION INTERFACE FORCES FOR MODEL 5
Fig. 6

USUAL WORK POINT LOCATIONS IN ECCENTRIC CONNECTIONS
Fig. 7
COMMONLY ASSUMED CONNECTIONS INTERFACE FORCES FOR ECCENTRIC CONNECTIONS
Fig. 12  SPECIMEN NO. 1, GROSS' TESTS
Fig. 13  SCHEMATIC OF TEST SPECIMEN
AND TEST FRAME, GROSS' TESTS
Fig. 14  MOMENT DIAGRAM FOR TEST 1A
AT 70k LATERAL LOAD
Fig. 15  THEORETICAL DISTORTION MOMENT DISTRIBUTION
Fig. 16

BEAM TO COLUMN FORCES EQUIVALENT TO 819k - in BEAM TO COLUMN MOMENT
Fig. 17

BEAM TO COLUMN CONNECTION FORCES TO SATISFY EQUILIBRIUM (TYPE III)
Fig. 18  DISTORTION AND EQUILIBRIUM FORCES COMBINED
Fig. 19  REDISTRIBUTION OF HORIZONTAL CONNECTION FORCES TO ACCOMMODATE CLIP ANGLE STRENGTH
Fig. 23  BRACING CONNECTIONS WITH ONE DIAGONAL PER JOINT
Fig. C1  TEST SPECIMENS FOR DOUTY AND McGUIRE TESTS
Fig. D1  GUSSET TEAROUT FAILURE MODES
APPENDIX A

Meeting - Joint ASCE/AISC Task Group
Heavy Bracing Connections
Kansas City, Missouri  March 13, 1990

Attendance:

J. Gross, Co-chairman  J. Wooten
W. Thornton, Co-chairman  R. Bjorhovde
D. Ricker  B. Vaughan
J. Griffiths  S. Fang
R. Disque  J. Ruddy (guest)
H. Astaneh

The following conclusions were agreed upon:

1. The load path through the gusset plate depends on the properties of the attachments (both stiffness and strength) and the choice of attachment (and therefore the attachment properties) depends on the load to be transferred. Thus, the design of gusset plate attachments is actually an indeterminate analysis/design problem.

2. The indeterminate analysis/design problem is very complex and not amenable to simplification.

3. The most promising "simple" approach is to return to basic principles including, but not limited to):

   a. satisfaction of static equilibrium
   b. recognition of the ductility of steel
   c. account of limits to ductility including buckling (instability) and possible brittleness in welds
   d. attention to good detailing practice (engineering judgement)
   e. economy of design

4. The task group recommends that attention be focussed on three methods which satisfy the above criteria:

   a. Model 2A - Engineering For Steel Construction Method
   b. Model 3 - Thornton method
   c. Model 4 - Parallel force (Ricker) method

5. These three methods reduce to two additional methods for the special case where the working point is "selected" to be located at the corner of the gusset plate.
6. All methods provide both a practical and safe solution to a complex problem while allowing a choice in the method used.

7. The "Design Guide" for heavy bracing connections should be organized as follows:
   a. description of the complexity of the problem (why solution defies a simple approach)
   b. discussion of "design philosophy" (rational approach) which satisfies the basic principles outlined above
   c. presentation of three methods including all equations and rationale as required
   d. examples of the application of these three methods to realistic problems - five worked solutions including three (one for each of the three methods) assuming a "concentric" attachment of the diagonal brace and two assuming an "eccentric" attachment of the diagonal brace.

8. Suggestions to designers/detailers in the following areas:
   a. good engineering practice
   b. reasonable assumptions of behavior
   c. avoidance of "pitfalls"
Appendix B

Typical Calculations
Chakrabarti/Bjorhovde
45° Case Using Model 3

\[ a = 4 + \frac{29}{2} = 18.5 \]
\[ b = 4 + 3 + 3 + \frac{11}{2} = 11.5 \]

Let \( \beta = b = 11.5 \)

\[ \alpha - \beta \tan \theta = e_B \tan \theta - e_C \]
\[ e_B = 12 \quad e_C = 6 \]
\[ \theta = 45 \quad \tan \theta = 1 \]
\[ \alpha = K + \beta \tan \theta \]
\[ K = e_B \tan \theta - e_C = 12 \times 1 - 6 = 6 \]
\[ \alpha = 6 + 11.5 = 17.5 \]

\[ r = \sqrt{(\alpha + e_C)^2 + (\beta + e_B)^2} = \sqrt{(17.5 + 6)^2 + (11.5 + 12)^2} = 33.2340 \]

\[ H_B = \frac{\alpha}{r} \quad P = \frac{17.5}{33.2340} \quad P = .5266P \]

\[ V_B = \frac{e_B}{r} \quad P = \frac{12}{33.2340} \quad P = .3611P \]

\[ V_C = \frac{\beta}{r} \quad P = \frac{11.5}{r} \quad P = .3460P \]

\[ H_C = \frac{e_C}{r} \quad P = \frac{6}{r} \quad P = .1805P \]

\[ M_B = V_B (\alpha - a) = .3611P (17.5 - 18.5) = -.3611P \]

\[ M_C = 0 \]

a. Brace to Gussett
   i. Whitmore
      \[ l_w = 18 \tan 30 \times 2 + 5 = 25.7846 \]
      \[ t_g = .125 \]

\[ F_y = 44 \text{ nominal} \quad F_y = 42.7 \text{ actual reported static} \]
\[ F_u = 65 \text{ nominal} \]
Use nominal values

\[ R_w = 25.7846 \times 0.125 \times 44 = 141.8^k \]

ii. Tearout

\[ A_{vg} = 19.25 \times 0.125 \times 2 = 4.8125 \]
\[ A_{vn} = (19.25 - 8.5 \times 0.875) \times 0.125 \times 2 = 2.9531 \]
\[ A_{tg} = 5 \times 0.125 = 0.6250 \]
\[ A_{tn} = (5 - 0.875) \times 0.125 = 0.5156 \]

\[ R_{TO1} = 4.8125 \times 0.6 \times 44 + 0.5156 \times 65 = 161 \]

127 (yield) 33.5 (fracture)

\[ R_{TO2} = 2.9531 \times 0.6 \times 65 + 0.6250 \times 44 = 143 \]

115 (fracture) 27.5 (yield)

\[ R_{TO} = 143^k \]

iii. Bolts \( \frac{3}{4}'' \) N-type

\[ F_v = 54 \text{ (LRFD)} \]
\[ r_v = 54 \times 0.4418 = 23.8 \]
\[ R_v = 18 \times 23.8 \times 2 = 857^k \]

iv. Bearing on gusset

\[ t_g = 0.125 \]
\[ t_w = 0.340 \]
\[ t_p = 0.375 \text{ ea.} \]

\[ P_b = 2.4 \times 65 \times 0.125 \times 0.75 \times 18 = 263^k \]

v. Other checks i.e. Brace tearout, bearing and splice plate bearing will not control.

vi. Splice Plate Net Section.

\[ A_g = 0.375 \times 7.5 \times 2 = 5.6250 \]
\[ A_n = 5.6250 - 2 \times 0.375 \times 0.875 \times 2 = 4.3125 \]
\[ 0.85 A_g = 4.7813 \text{ use 4.3125} \]
\[ P_g = 5.6250 \times 44 = 247.5^k \]
\[ P_n = 4.3125 \times 65 = 280.3^k \]
b. Gusset to Beam

i. Plate yield

\[ f_v = \frac{H_B}{.125 \times 29} = .6 \times 44 = 26.4 \]

\[ f_v = \frac{.5266P}{.125 \times 29} = 26.4 \quad P = 181.7 \]

\[ f_t = \frac{V_B}{.125 \times 29} = 44 \]

\[ = \frac{.3611P}{.125 \times 29} = 44 \quad P = 441.7 \]

\[ f_b = \frac{6M_B}{.125 \times 29^2} = 44 \]

\[ = \frac{6 \times .3611P}{.125 \times 29^2} = 44 \quad P = 2135 \]

\[ f_t + f_b = \frac{.3611P}{.125 \times 29} + \frac{6 \times .3611P}{.125 \times 29^2} = 44 \]

\[ .0966P + .0206P = 44 \quad P = 366^k \]

ii. Weld \( \frac{1}{8} F_w \)

\[ f_a = (0.0996 + 0.0206) P \times \frac{125}{2} = .0075P \]

\[ f_v = .1453P \times \frac{125}{2} = .0091P \]

\[ \sqrt{.0075^2 + .0091^2} \times P = 1.856 \times 2 \]

\[ P = 315^k \]

c. Gusset to Column

i. Net Shear

\[ A_V = (25.75 - 6 \times .875) \times .125 = 2.5625 \]

\[ R_{ns} = 2.5625 \times .6 \times 65 = 99.9^k \]
ii. Block Shear

\[ A_{vg} = 19 \times .125 = 2.3750 \]
\[ A_{vn} = (19 - 5.5 \times .875) \times .125 = 1.7734 \]
\[ A_{tg} = 2.25 \times .125 = .2813 \]
\[ A_{tn} = (2.25 - .5 \times .875) \times .125 = .2266 \]
\[ R_{BS1} = 2.3750 \times .6 \times 44 + .2266 \times 65 = 77.4 \]
\[ R_{BS2} = 1.7734 \times .6 \times 65 + .2813 \times 44 = 81.5 \]
\[ R_{BS} = 81.5 < R_{NS} = 99.9 \]

Block Shear Controls

iii. Tension Tearout

\[ A_{vg} = 2.25 \times .125 \times 2 = .5625 \]
\[ A_{vn} = (2.25 - .5 \times .875) \times .125 \times 2 = .4531 \]
\[ A_{tg} = 15 \times .125 = 1.8756 \]
\[ A_{tn} = (15 - 5.5 \times .875) \times .125 = 1.2734 \]
\[ R_{T1} = .5625 \times .6 \times 44 + 1.2734 \times 65 = 97.6 \]
\[ R_{T2} = .4531 \times .6 \times 65 + 1.8750 \times 44 = 100.2 \]
\[ R_{T} = 97.6 \]
iv. Couple Tearout - none this case but capacity calculated as follows:

\( \mu_c = .6 F_u \left( e - \frac{1}{2} d \right) (n - 1) \text{ pt} + F_y p \left( \frac{n - 1}{2} \right)^2 \text{ pt} \)

or...

\( = .6 F_y e \left( n - 1 \right) \text{ pt} + F_u \left( p - d \right) \left( \frac{n - 1}{2} \right)^2 \text{ pt} \)

\[
\mu_c = .6 \times 65 \times \left( 2.25 - \frac{.875}{2} \right) \times 5 \times 3 \times .125
\
132.5 \ (F)
\
+ 44 \times 3 \times \left( \frac{5}{2} \right)^2 \times 3 \times .125
\
309.3 \ (Y)
\
= 442 \ k\text{-in}

\[
\mu_c = .6 \times 44 \times 2.25 \times 5 \times 3 \times .125
\
111.4 \ (Y)
\
+ 65 \times (3 - .875) \times \frac{25}{4} \times 3 \times .125
\
313.8 \ (F)
\
= 425 \ k\text{-in}

\[
\mu_c = 425 \ k\text{-in}
\]

Interaction

\[
\frac{V_c}{81.5} + \frac{H_c}{97.6} + \frac{M_c}{(425)} = 1
\]

\[
\frac{.3460P}{81.5} + \frac{.1805}{97.6} = 1
\]

\[ P = 164k \]
v. Bearing on Critical Bolt

\[
\frac{V_c}{2} \rightarrow \frac{H_c}{2} \\
\sqrt{\left(\frac{V_c}{6}\right)^2 + \left(\frac{H_c}{6}\right)^2} = 2.4 \times 65 \times .125 \times .75 \\
\sqrt{(.3640P)^2 + (.1805P)^2} = 6 \times 2.4 \times 65 \times .125 \times 7 \\
.4063P = 87.8
\]

\[P = 216^k\]

vi. Bolt Shear

\[.4063P = 6 \times 23.8\]

\[P = 351^k\]

vii. Clip Angles

\[
b = 2\frac{1}{4} - .375 = 1.8750 \quad b' = 1.50
\]
\[
a = 3.5 - 2.25 = 1.25 \quad a' = 1.625
\]
\[
\delta = 1 - \frac{.8125}{3} = .7292
\]

Ult. strength: \(B_u = 90 \times .4418 = 39.8^k\)

\[F_u = 65\]
\[ \alpha_u = \frac{1}{\delta (1 + \rho)} \left[ \frac{4B_d b'}{pt_f^2 F_u} - 1 \right] \]

\[ = \frac{1}{0.7292 \times 1.9231} \left[ \frac{4 \times 39.8 \times 1.50}{3 \times 0.375^2 \times 65} - 1 \right] = 5.4969 \]

\[ T_u = \frac{3 \times 0.375^2 \times 65}{4 \times 1.5} \{1 + 0.7292\} = 7.9030^k/\text{Bolt} \]

Max Applied Tension per Bolt = \( \frac{1805P}{12} = .0150P \)

\[ P = \frac{7.9030}{.0150} = 525^k \]

Verify that no shear interaction occurs @ \( P = 525 \)

Shear per Bolt = \( \frac{V_c}{12} = \frac{.3460 \times 525}{12} = 15.1492^k \)

\[ f_t = \frac{F_t}{F_v} \sqrt{F_v^2 - f_v^2} \]

\[ = 1.67 \sqrt{F_v^2 - f_v^2} \]

\[ F_v = 54 \]

\[ f_v = 34.29 \]

\[ f_t = 1.67 \sqrt{54^2 - 34.29^2} = 69.66 \]

\[ B_u = 69.66 \times .4418 = 30.8 \]

\[ \alpha_u = \frac{1}{0.7292 \times 1.9231} \left[ \frac{4 \times 30.8 \times 1.5}{3 \times 0.375^2 \times 65} \right] = 4.0926 \]

\( \alpha_u \) still > 1 \( T_u = 7.9030^k/\text{Bolt} \)

d. Beam to Column

Shear = \( V_B = .3611P \)

Axial = \( H_B = .5266P \) because of transfer force to floor or test frame.
i. Net Shear

\[ A_vn = (20.75 - 7 \times .875) \times .468 = 6.8449 \]
\[ R_{ns} = 6.8449 \times .6 \times 65 = 266.9^k \]

ii. Block Shear - about same as net shear.

iii. Tension Tearout

\[ R_{TO} = 6.8449 \times 65 = 444.9 \]
\[ \frac{.3611P}{266.9} + \frac{.5266P}{444.9} = 1 \]
\[ P = 394^k \]

iv. Bolt Shear

\[ \sqrt{(\.3611P)^2 + (.5266P)^2} = 7 \times 23.8 \times 2 \]
\[ P = 522^k \]

v. Bearing

\[ .6385P = 7 \times 2.4 \times 6.5 \times .468 \times .75 \]
\[ P = 600^k \]
vi. Bending of Clips

\[ T_u = 7.9030 \]

\[ \frac{0.5266P}{14} = 7.9030 \]

\[ P = 210^k \]

SUMMARY:

- Brace to Gusset  \( 142^k \)  Whitmore, Tearout
- Gusset to Beam  \( 182^k \)  Gusset Yield
- Gusset to Column  \( 164^k \)  Tearout
- Beam to Column  \( 210^k \)  Bending of Clips
Appendix C

Method To Evaluate The Strength Of Double Angles In Tension

The test specimens of Chakrabarti and Bjorhovde\(^2\) and of Gross\(^3\) all use double angles to connect the gussets and the beam to the column flange or web. In order to evaluate the test results, a reliable method to assess the yield strength and the ultimate breaking strength is needed. As a candidate method, consider the method of Fisher and Struik\(^5\), as presented in the AISC Manual, 9th Edition\(^6\) as formulated by Thornton\(^7\).

This method can be written as follows for yield strength:

\[
\alpha_y = \frac{1}{\delta(1+\rho)} \left[ \frac{4B_y b}{pr^2 F_y} \cdot 1 \right]
\]

If \(\alpha_y \leq 0\), \(T_y = B_y\)

If \(0 < \alpha_y \leq 1\), \(T_y = \frac{pr^2 F_y (1 + \delta \alpha_y)}{4b}\)

(if \(\alpha_y > 1\), set \(\alpha_y = 1\))

For ultimate strength

\[
\alpha_u = \frac{1}{\delta(1+\rho)} \left[ \frac{4B_u b}{pr^2 F_u} \cdot 1 \right]
\]

If \(\alpha_u \leq 0\), \(T_u = B_u\)

If \(0 < \alpha_u \leq 1\), \(T_u = \frac{pr^2 F_u (1 + \delta \alpha_u)}{4b}\)

(if \(\alpha_u > 1\), set \(\alpha_u = 1\))
In the above equations, the quantities $\alpha$, $\delta$, $p$, $b'$, $t$, $F_Y$, $F_U$, $B$ and $T$ are as defined in the AISC Manual, 9th Edition, except that $B_Y$ is the bolt yield strength, $B_U$ is the bolt ultimate strength, $T_Y$ is the yield load per bolt for the bolt-flange combination and $T_U$ is the ultimate strength per bolt of the bolt flange combination.

To test these formulations, the test data produced by Douty and McGuire\(^8\) for T stubs are used. Although double clips are not T stubs, it has been common practice for many years to treat T stubs and double clips in a similar manner. Fig.C1 shows the test specimens and Table C1 gives the Douty and McGuire geometric and material data. Table C2 gives the results of applying the above yield and ultimate strength formulation to the specimens of Fig. C1. Table C2 also gives the actual (experimental) yield load, ultimate load, and actual failure made. In Table C2, $P_Y = 4T_Y$ and $P_U = 4T_U$, i.e., $P_Y$ and $P_U$ are the assembly yield and ultimate strengths which can be compared directly with the actual (experimental) strengths. Table C3 gives a direct comparison between actual and theoretical results. It can be seen from Tables C2 and C3 that the theory gives excellent agreement with the test results for ultimate strength and generally very good results for yield strength. Some observations on these results can be made. Concerning the ultimate strength results, it is pointed out by Kato and McGuire\(^9\) that after the formation of the collapse mechanism in the T flange (i.e., when the plastic bending stress is $F_U$ at both the bolt line and the stem line) the T stub can sustain the load by "hanging action". On the contrary, the post yield strength of the high strength bolts is small and the behavior is less ductile. Thus, in the tests, bolt fracture is usually the failure mode even if the theory indicates that the flange or web will fail first. Nevertheless, the significant deformations of the flange that take place at the $P_U$ load do cause additional load to be placed in the bolts, and this additional load (conventionally called "prying action") does cause the bolts to fracture before their nominal ultimate strength, i.e., $4B_U$, is achieved when the flange is flexible, i.e. $\alpha < 0$.

It will be observed from Tables C2 and C3 that the agreement between experiment and theory for yield is not as good as that for ultimate strength. This occurs because yielding is defined, following Kato and McGuire, as the load at which the bolts first "see" load in addition to the pretension load. This point is also called the "separation point". In two instances in Table C2, Kato and McGuire reported that the separation point was not clear (Tests A11 and A15) and no yield load is given. In general, because there is no clear catastrophic failure, the yield load would be much more dependent on small variations in material properties, thickness, initial pretension, and the like. Therefore, it is reasonable to find a bigger variation between theory and experiment here than in the ultimate strength results. On the whole, it is felt that the agreement between theory and experiment for the yield results is very good.

As a final note on the ultimate strength formulation proposed above, the idea for using $F_U$, the ultimate strength of the T stub material, as the limiting bending stress in the T stub flange, is due to Kato and McGuire\(^9\). The excellent agreement shown in Table C3 for actual/theoretical ultimate strength validates this idea.
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<td>d (in)</td>
<td>a (in)</td>
<td>b (in)</td>
<td>a' (in)</td>
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*indicates data based on non-rigid base.
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<th>Test No.</th>
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<th>Actual Strength (Experimental)</th>
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(1) Based on $F_y = 26.0$ in T stub
(2) Based on $F_y = 31.0$ in T stub
(3) Imperfection in material
(4) Exceeded machine capacity of 404 kips
(5) No Abrupt increase bolt tension. Separation point (yield point) is not clear.

* indicates data based on non-rigid base
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Appendix D

Tearout Under Shear, Tension and Couple

When considering the forces at the edge of a gusset, it is possible to have shear, tension, and a couple all acting simultaneously. It is necessary to have a method to assess the strength of such a situation with respect to a tearout type of failure. Tests have been performed to determine strength under shear alone or tension alone, but no tests have been performed with these two acting simultaneously. No tests have been performed to determine strength under couple, either alone or in combination with shear and tension. Thus it is necessary to postulate a strength criterion for this situation. Because of the lack of data, a straight line interaction strength criterion is proposed as,

\[
\frac{V}{V_u} + \frac{T}{T_u} + \frac{M}{M_u} \leq 1
\]

where \( V_u \) is the shear capacity,
\( T_u \) is the tensile capacity
\( M_u \) is the couple capacity

A straight line interaction criterion is suggested, as mentioned, because of a lack of data, but also because it is generally recognized as being conservative.

Figure D1 shows the possible failure modes for each of the three forces acting separately. Formulas to compute capacities \( V_u \) and \( T_u \) can be found in the commentary to the AISC LRFD Specification.(10)

A formula for couple strength is as follows:

\[
M_u = \max \left\{ 0.6F_u \left( e - \frac{1}{2} d' \right) \left( n - 1 \right) pt + F_y p \left( \frac{n-1}{2} \right)^2 pt, \ 0.6F_y e \left( n - 1 \right) pt + F_u \left( p - d' \right) \left( \frac{n-1}{2} \right)^2 pt \right\}
\]

where:
- \( e = \) edge distance
- \( p = \) bolt spacing
- \( d' = \) hole diameter + \( \frac{1}{16} \)
- \( n = \) number of bolts
- \( t = \) gusset plate thickness