A MEMBRANE METHOD FOR TRANSVERSELY LOADED COLUMN WEBS

William A. Thornton
President, Cives Engineering Corporation, Roswell, Georgia, USA

ABSTRACT

Existing methods for estimating the nominal strength of column webs subjected to loads delivered by plates parallel to the column flanges and centered on the web are based on either elastic beam or plate theory, or on plastic plate (yield line) theory. These methods have been shown by physical testing to greatly underestimate the nominal strength. A method based on non-linear membrane action has been developed and compared with the results of physical tests. Excellent correlation between the actual and predicted nominal strengths has been achieved. The development of the method, comparison to physical tests, and application to a design example will be presented.

INTRODUCTION

The basic problem is shown in Fig. 1. The plate is welded to the center of the column web as shown, and is loaded transverse to the column web. This problem occurs with knee braces and other bracing situations.

There are two solutions available in the literature. Blodgett (1) gives an elastic solution based on beam theory. Anand and Bertz (2) present a yield line solution, although they have access to a series of physical tests which show that the yield line solution is very conservative and that membrane action is necessary to predict the actual behavior.

If the recommended elastic method (1) or the yield line method (2) are used, the capacity of this connection will be very small and stiffeners such as those shown in Fig. 2 will almost always be necessary. Because this is an expensive solution, it is the purpose of this paper to develop a membrane method which more accurately predicts the load carrying capability of this connection.

THEORY

The membrane model is shown in Fig. 3. The central portion of length l, will pull up under a tension load as a 2-link mechanism. Following Timoshenko and Young (3), the tensile force P_u will be given by

$$P_u = 2\sqrt{2}t_w l\sqrt{\frac{F_y^{\ 3}}{E}} \qquad (1)$$

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when the links reach their ultimate strength of $F_y t_w$ per unit length, where F_y is the yield stress and t_w is the column web thickness. Eq. 1 is the basic membrane formula. The web displacement at the load P_u is

$$d = \frac{1}{\sqrt{2}}\sqrt{\frac{F_y}{E}}h \qquad (2)$$

One advantage of the membrane method over a yield line method is the capability to calculate a displacement as in eq. 2. This equation will assist in arriving at a reasonable failure criterion for this connection.

Returning to eq. 1, which represents only the central portion of the displacement pattern shown in Fig. 3, consider that the circular end portions of the pattern will also add some capacity. Taking the average length of the radial links as their mid length, the l in eq. 1 is replaced by

$$l_T = l + \frac{ap}{2} = l + \frac{hp}{4}$$

Thus eq. 1 becomes

$$P_u = 2\sqrt{2}t_w l_T \sqrt{\frac{F_y{}^3}{E}} \qquad (3)$$

A radial fan yield line can exist perpendicular to the link lengths, and from Park and Gamble (4), the extra capacity is

$$2pm_p = 2p\,\frac{1}{4}F_y t_w{}^2 = \frac{p}{2}F_y t_w{}^2$$

So eq. 3 becomes

$$P_u = t_w F_y\left[2\sqrt{2}\sqrt{\frac{F_y}{E}}l_T + \frac{p}{2}t_w\right] \qquad (4)$$

Referring again to Fig. 3, if l is too long, the column flanges will not be able to sustain the pull of the links of $t_w F_y$ per unit length and will tend to become closer together. To prevent this happening at the load P_u, consider the length of flange noted by u in Fig. 3. Considering Fig. 4, the flange will be able to sustain the pull $t_w F_y$ as long as

$$t_w F_y u\,\frac{u}{2} \le 2\frac{1}{4}F_y b_f t_f{}^2 \qquad (5)$$

where b_f and t_f are the column flange width and thickness, respectively. Then, from eq. 5, if

$u = t_f \sqrt{\dfrac{b_f}{t_w}}$ and $l \le 2u$, the original model can be sustained. Setting l_T of eq. 4. equal to $l_{eff} + \dfrac{p}{4}h$

where $l_{eff} = \min\{l, 2u\}$ (6)

and recognizing that if l_{eff} is less than l, the portion of the pattern in the middle, i.e., $l - l_{eff}$ is no longer sustaining membrane action but can still develop yield line strength $\left(l - l_{eff}\right)\dfrac{1}{4}F_y t_w^{\,2}(4)$, eq. 4. is finally completed as

$$P_u = t_w F_y \left[2\sqrt{2}\sqrt{\frac{F_y}{E}}(l_{eff} + \frac{p}{4}h) + \frac{t_w}{h}(\frac{p}{2}h + l - l_{eff})\right] \qquad (7)$$

This is the proposed membrane method equation for transversely loaded webs.

EXPERIMENTAL VERIFICATION

A large number of laboratory tests were performed by Csernak (5) on this connection. His data are presented in Table 1. All material used was A36, but only one coupon test was reported in (5), although several were reported to have been done. The material was definitely established as A36. The reported properties of F_y and E were 36.5 ksi and 31,600 ksi, respectively. Because of the lack of data, Table 2 was produced with F_y = 36 ksi and E = 29,000 ksi. The $\left(P_u\right)$ test in Table 2 was determined from the load-deflection curves of (5). The value of deflection was calculated from eq. 2. At this value of d, the value of $\left(P_u\right)$ test was determined from the load-deflection curve and listed in Table 2. The value of $\left(P_u\right)$ theory was calculated from eq. 7. The last column of Table 2 shows a fairly good agreement of the "test" and "theory" values of $\left(P_u\right)$. The statistical error in any one observation in the last column of Table 2 is 11.9%, which gives some confidence that the predicted load capacity (nominal strength) of this connection can be determined to within ±12%, which is satisfactory for engineering calculations. From Table 2, it can be seen that the deflection at the nominal strength d is generally smaller than the column web thickness. Since this a reasonable displacement for most structures, the nominal strength (or ultimate load) $\left(P_u\right)$ predicted by eq. 7 is a reasonable strength prediction.

AN EXAMPLE (From 6)

Consider a W16x50 column of A572-50 steel with a plate ½x24x18 of A36 steel welded to the center of its web. What design load can this configuration carry? See Fig. 1.

For the column, d = 16.26, t_f = .630, h = 16.26 – 2 x .630 = 15.0, b_f = 7.070, t_w = .380;

For the plate, l = 24. Then

$$u = t_f \sqrt{\frac{b_f}{t_w}} = .630\sqrt{\frac{7.070}{.380}} = 2.717$$

$$l_{eff} = \min\{2x2.717, 24\} = 5.44$$

$$P_u = .380x50\left[2\sqrt{2}\sqrt{\frac{50}{29{,}000}}\left(5.44 + \frac{p}{4}x15.0\right) + \frac{.380}{15.0}\left(\frac{p}{2}x15.0 + 24 - 5.44\right)\right] = 58.7kips$$

and the design strength is $fP_u = .9x58.7 = 52.8^k$

For comparison, a yield line solution (2) is $P_u = 2t_w^{\ 2}F_y\left(\sqrt{2} + \frac{1}{2}\frac{l}{h}\right)$ which will yield

$P_u = 2x.380^2x50\left(\sqrt{2} + \frac{1}{2}x\frac{24}{15}\right) = 31.9$ kips and $fP_u = .9x31.9 = 28.8$ kips, which is about half of the membrane solution for this case.

CONCLUSIONS

A membrane theory method for transversely loaded column webs has been developed and shown to be validated by physical tests. The method generally gives greater capacities than the usual yield line method with displacements which are on the order of the column web thickness.

REFERENCES

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Table 1 Test Specimens (from Csernak 6)

Test No.	Column Type	Plate Size
1	W6 x 8.5	4 x 3/8
2	W6 x 15.5	8 x 3/8
3	W6 x 16	4 x 3/8
4	W6 x 16	8 x 3/8
5	W6 x 25	4 x 3/8
6	W8 x 10	8 x 3/8
7	W8 x 24	4 x 3/8
8	W8 x 24	6 x 3/8
9	W8 x 24	8 x 3/8
10	W8 x 31	2 x 3/8
11	W8 x 31	4 x 3/8
12	W8 x 31	6 x 3/8
13	W8 x 31	8 x 3/8
14	W8 x 31	8 x 3/8
15	W10 x 11.5	8 x 3/8
16	W10 x 21	4 x 3/8
17	W10 x 21	4 x 3/8
18	W10 x 21	4 x 3/8
19	W10 x 21	8 x 3/8
20	W10 x 33	8 x 3/8

Table 2 Comparison of Physical Test Data With the Theory

Test no.	l in.	$2u$ in.	h in.	l_{eff} in.	t_w in.	$(P_u)test$ kips	d in. Eq. 2	$(P_u)theory$ kips Eq. 7	$\frac{(P_u)test}{(P_u)theory}$
1	4	1.78	5.54	1.78	.175	6.02	.138	5.2	1.16
2	8	2.69	5.56	2.69	.240	11.3	.138	15.5	.73
3	4	3.15	5.44	3.15	.265	11.4	.136	13.0	.88
4	8	3.15	5.44	3.15	.265	13.3	.136	16.0	.83
5	4	3.98	5.42	3.98	.325	16.6	.136	19.3	.86
6	8	1.97	7.46	1.97	.180	7.83	.186	7.0	1.12
7	4	3.89	7.20	3.89	.275	13.7	.179	15.5	.88
8	6	3.89	7.20	3.89	.275	14.5	.179	18.2	.80
9	8	3.89	7.20	3.89	.275	15.2	.179	17.1	.89
10	2	4.35	7.28	2	.305	13.7	.181	15.2	.90
11	4	4.35	7.28	4	.305	15.9	.179	18.8	.85
12	6	4.35	7.28	4.35	.305	18.0	.181	21.5	.84
13	8	4.35	7.28	4.35	.305	18.0	.181	26.0	.69
14	8	4.35	7.13	4.35	.295	17.1	.178	22.5	.76
15	8	1.89	9.60	1.89	.180	8.66	.239	6.5	1.33
16	4	3.36	9.31	3.36	.245	12.9	.232	10.3	1.25
17	4	3.36	9.31	3.36	.245	12.9	.232	10.3	1.25
18	4	3.36	9.31	3.36	.245	12.9	.232	10.3	1.25
19	8	3.36	9.31	3.36	.245	12.9	.232	14.4	.90
20	8	4.63	8.99	4.63	.295	18.5	.224	20.0	.93

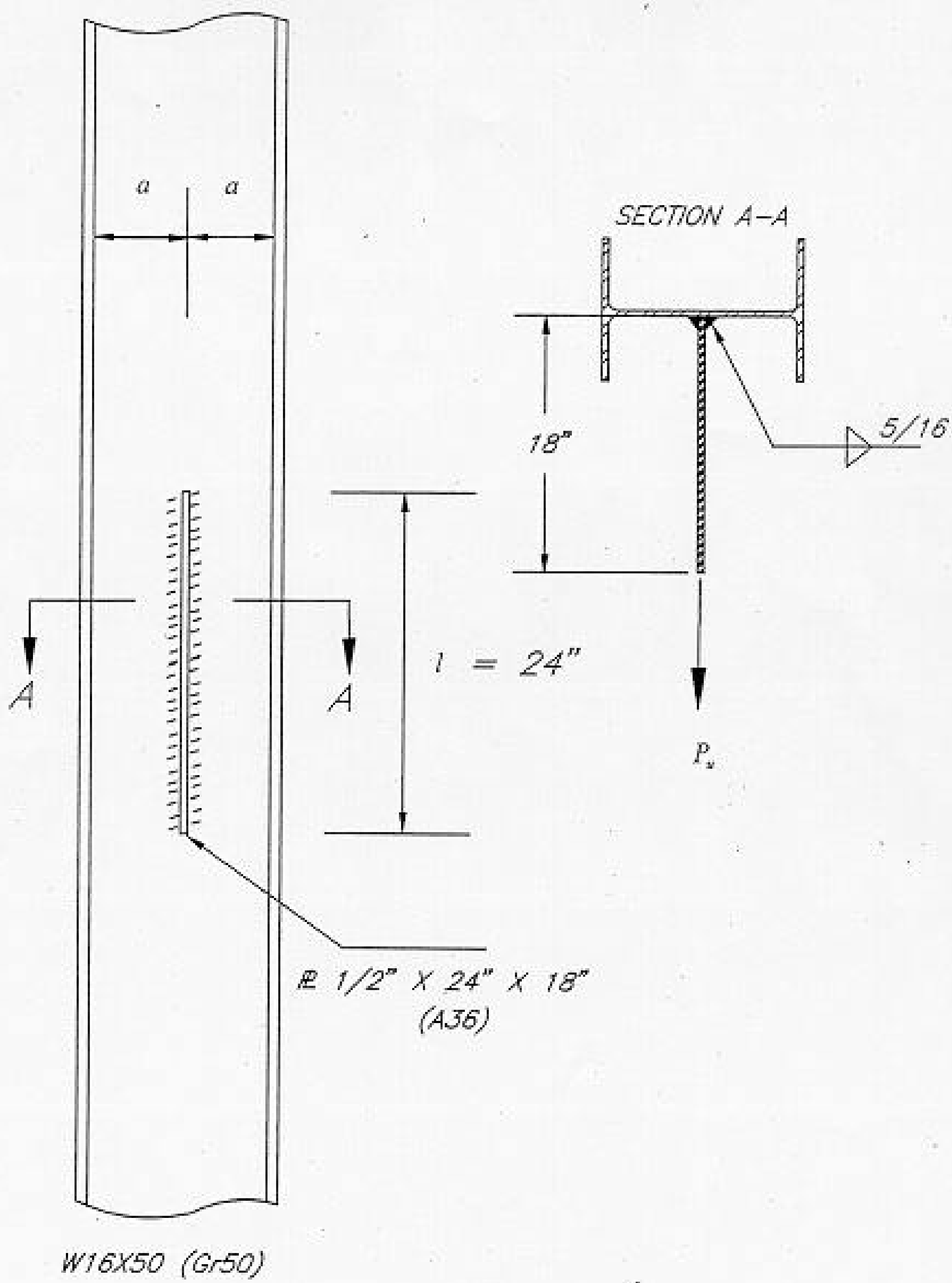

FIG. 1 TRANSVERSELY LOADED COLUMN WEB (FROM 6)

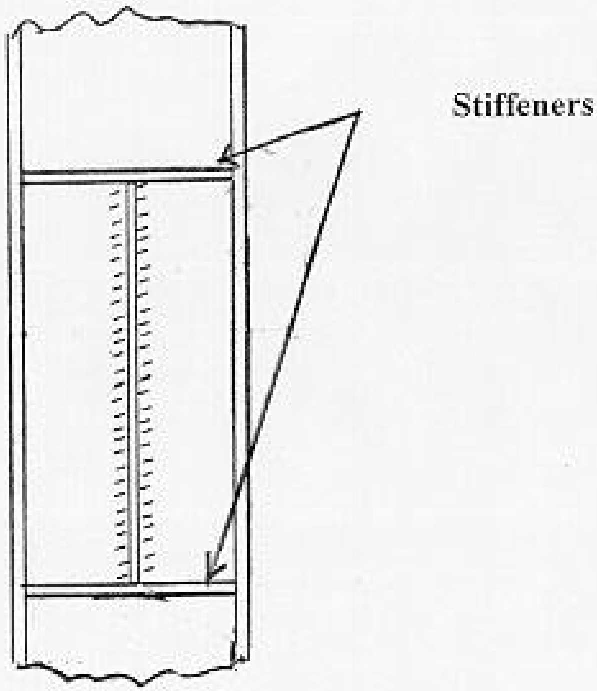

FIG 2. STIFFENERS MAY BE REQUIRED TO CARRY LOAD TO COLUMN FLANGES

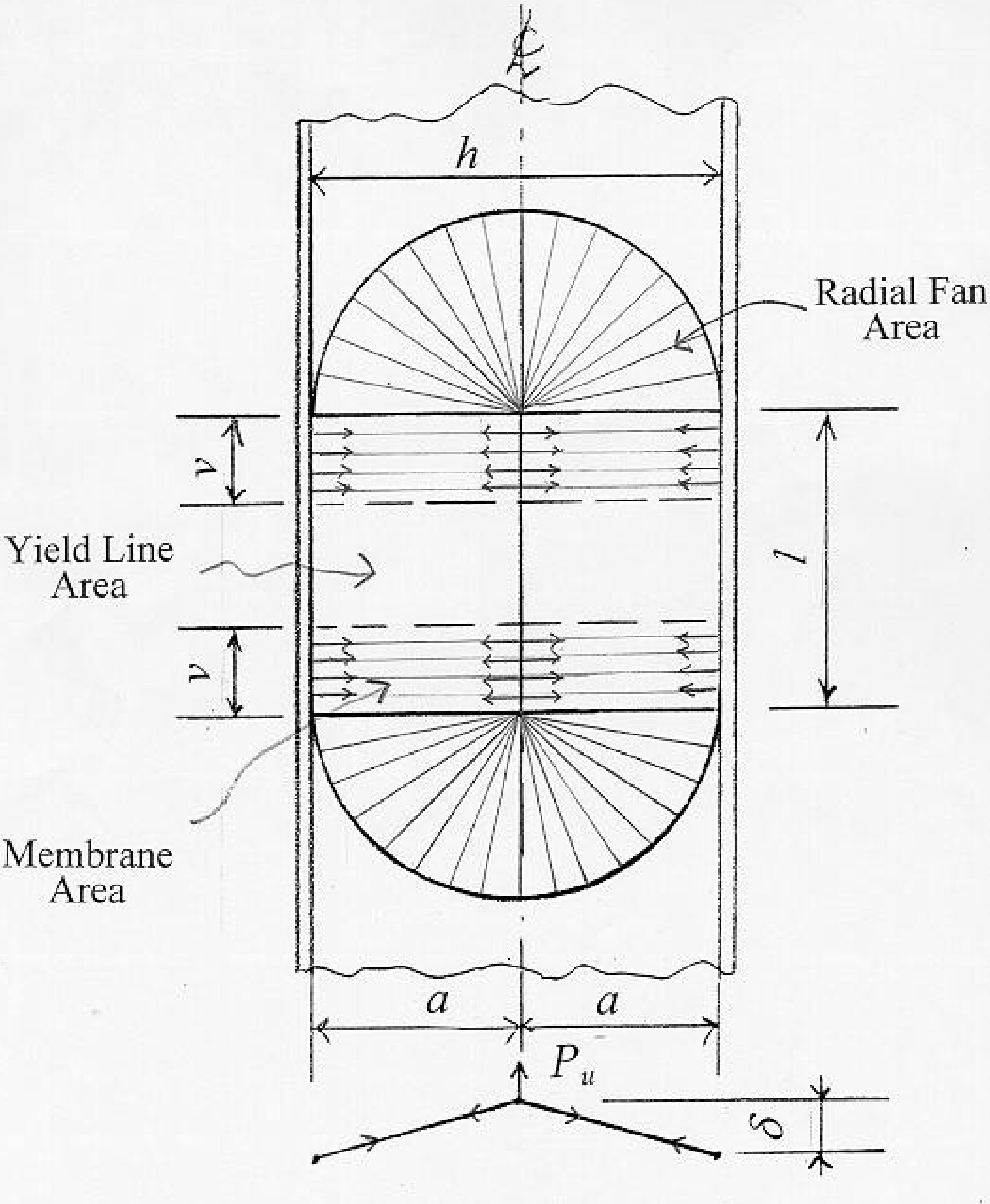

FIG. 3 MEMBRANE MODEL

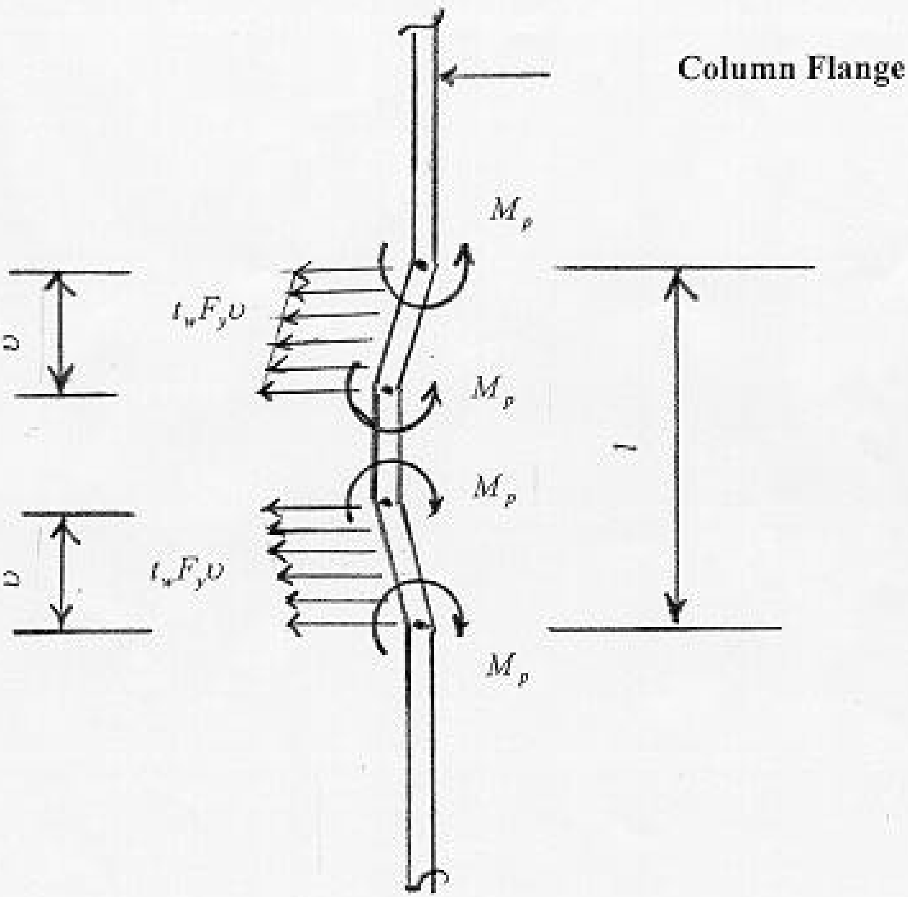

FIG. 4 FLANGE MECHANISM INDUCED BY WEB MEMBRANE FORCES